Grade 7/8 Math Circles

March 4 & 5 & 6 & 7, 2024

The Bayesian Clinical Diagnostic Model - Problem Set

1. Let's review some terminology.

   (a) Which of the following test results could you receive if you actually have the disease (*circle some of the terms below*)?

   false positive   false negative   true positive   true negative

   (b) Which of the following test results could you receive if you don't actually have the disease (*circle some of the terms below*)?

   false positive   false negative   true positive   true negative

   (c) If you actually have the disease, the chance that the diagnostic test correctly detects that you have the disease is called (*circle one of the terms below*):

   specificity   sensitivity

   (d) If you don't actually have the disease, the chance that the diagnostic test correctly detects that you don't have the disease is called (*circle one of the terms below*):

   specificity   sensitivity

   (e) Another name for the specificity of a test is: ____________________

   (f) Another name for the sensitivity of a test is: ____________________

2. Check your understanding.

   (a) In a fixed population of people who either have a disease or don't, increasing the number of true negatives will definitely decrease which of the below?

   false positive   false negative   true positive   true negative

   (b) In a population of people who either have a disease or don't, a test that yields a high number of true positives relative to false negatives has a high __________ (*circle one of the terms below*).

   specificity   sensitivity

   (c) Which of the two properties of a test can be important in reducing unnecessary psychological impacts of social stigma (*circle the best answer below*)?

specificity   sensitivity

(d) Which quantity or quantities are influenced by the prevalence of disease in a given population?

specificity   sensitivity   positive predictive value

(e) Suppose that the prevalence of a disease is low (about 1%). Suppose that the specificity of a test is 75% and that the sensitivity of the test is 90%. Which of the following adjustments to the positive predictive value would result in the largest increase to the positive predictive value?

i. Raising the sensitivity of the test to 100% and keeping the specificity of the test at 90%.

ii. Keeping the sensitivity of the test at 75% and raising the specificity of the test to 100%.

3. A diagnostic test results in

- 5 true positives
- 2 false positives
- 8 true negatives
- 3 false negatives

Based solely on these results, calculate the most probable

(a) prevalence of the disease

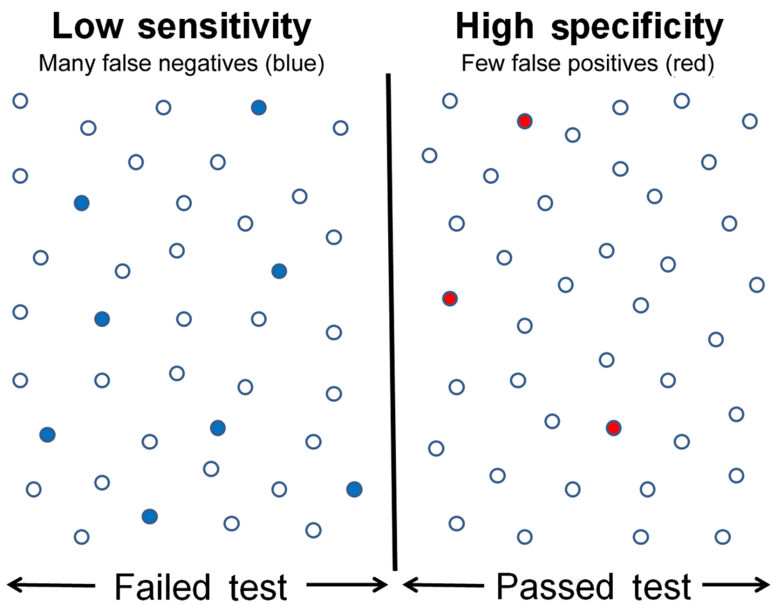
(b) sensitivity of the test

(c) specificity of the test

(d) positive predictive value of the test

4. Interpret the following diagram[1] of people who have passed or failed a diagnostic test. Calculate the sensitivity and specificity of the test based on the results shown in the diagram.

[1]**Diagram source**: Rmostell, CC0, via Wikimedia Commons



5. The sensitivity of a test is 50% and the specificity of the test is 80%. In a group of 1000 people who are tested, 50 actually have the disease.
   (a) What is the prevalence of the disease?
   (b) Predict how many of the 1000 people tested are true positives.
   (c) Predict how many of the 1000 people tested are false positives.
   (d) Predict how many of the 1000 people tested have a positive test result (either a true positive or a false positive).
   (e) What is the positive predictive value of the test?
6. Suppose that a test for a certain condition has a sensitivity of 100% and a specificity of 95%. Out of 1000 people tested, 145 test positive and the rest test negative. How many true positives, false positives, true negatives and false negatives are there expected to be?
7. Test A is performed on exactly 1 individual who actually has the disease, returning a true positive test result and exactly 2 individuals who don't actually have the disease, both returning a true negative test result. Test B is performed on 360 individuals who actually have the disease and 370 individuals who don't actually have the disease. When the test results come back, 349 of the 360 individuals who have the disease receive a positive test result, with the rest receiving a negative test result, and 350 out of the 370 individuals who don't actually have the disease receive a negative test result, with the rest receiving a positive test result.
   (a) Calculate the sensitivity and specificity for each test.

(b) Based solely on this data, which test is less risky to use? Explain your reasoning.

8. The *false positive rate* (FPR) or *fallout* of a test is the probability that someone without the disease is not correctly identified by the test (i.e., they receive a false negative).

   (a) Suppose that a test has a sensitivity of 30% and a specificity of 96%. What is the false positive rate of the test?

   (b) Suppose that a test results in 15 false positives and 60 true negatives. What is the false positive rate of the test?

   (c) For a test of a certain ailment, 5 out of every 50 people who don't have the ailment test positive. What is the false positive rate of the test?

   (d) The prevalence of a disease is 1 in 1000. The false positive rate of a test for the disease is 5%, and the test never fails to detect someone who really has the disease. If someone's test result comes back positive, what are the chances that they actually have the disease?